Here the ODE is $-y'' = 2y + x$. $(\mu = 2)$

Consider the BVP $y'' + \lambda y = 0,\ y(0) = 0,\ y'(1) + y(1) = 0$.

The normalized eigenfunctions are (see website)

$$\phi_n(x) = \frac{\sqrt{2}}{\sqrt{1+\cos^2\sqrt{\lambda_n}}} \sin(\sqrt{\lambda_n}\, x)$$

where $\lambda_n \approx (2n-1)^2 \frac{\pi^2}{4}$, $n \geq 1$, $\sqrt{\lambda_n}$ satisfies $\sqrt{\lambda_n}\cos\sqrt{\lambda_n} + \sin\sqrt{\lambda_n} = 0$.

Next we want to express $f(x) = x$ in terms of $\phi_n(x)$:

$$c_n = \int_0^1 x \cdot k_n \cdot \sin(\sqrt{\lambda_n}\, x)\, dx \qquad \left(k_n = \frac{\sqrt{2}}{\sqrt{1+\cos^2(\sqrt{\lambda_n})}}\right)$$

$$= k_n \int_0^1 x \sin(\sqrt{\lambda_n}\, x)\, dx \qquad \lambda_n \approx (2n-1)^2 \frac{\pi^2}{4}.$$

$$= k_n \left[-\frac{x\cos\sqrt{\lambda_n}\,x}{\sqrt{\lambda_n}} \Big|_0^1 + \int_0^1 \frac{\cos\sqrt{\lambda_n}\,x}{\sqrt{\lambda_n}}\, dx \right]$$

$u = x$, $du = 1\,dx$; $dv = \sin\sqrt{\lambda_n}\,x\, dx$, $v = -\frac{\cos\sqrt{\lambda_n}\,x}{\sqrt{\lambda_n}}$

$$= k_n \left[-\frac{\cos\sqrt{\lambda_n}}{\sqrt{\lambda_n}} + \frac{1}{\lambda_n}\left[\sin\sqrt{\lambda_n}\,x\right]_0^1 \right]$$

$$= k_n \left[-\frac{\cos\sqrt{\lambda_n}}{\sqrt{\lambda_n}} + \frac{1}{\lambda_n}\sin\sqrt{\lambda_n} \right]$$

$$= k_n \left[\frac{-\sqrt{\lambda_n}\cos\sqrt{\lambda_n} + \sin\sqrt{\lambda_n}}{\lambda_n} \right] = \frac{k_n}{\lambda_n} \cdot -2\sqrt{\lambda_n}\cos\sqrt{\lambda_n}$$

$$\Rightarrow c_n = -\frac{2k_n}{\sqrt{\lambda_n}}\cos\sqrt{\lambda_n} = -\frac{2\sqrt{2}\cos\sqrt{\lambda_n}}{\sqrt{\lambda_n}\sqrt{1+\cos^2\sqrt{\lambda_n}x}}$$

$\left(\sqrt{\lambda_n}\cos\sqrt{\lambda_n} + \sin\sqrt{\lambda_n} = 0 \Rightarrow \sin\sqrt{\lambda_n} = -\sqrt{\lambda_n}\cos\sqrt{\lambda_n}\right)$

Hence the solution of the nonhomogeneous BVP is

$$y = \sum_{n=1}^{\infty} b_n \phi_n(x)$$

where $\phi_n(x)$ is given above, and $b_n = \frac{c_n}{\lambda_n - 2}$, c_n's are above and $\lambda_n \approx \frac{(2n-1)^2\pi^2}{4}$, $n \geq 1$.