Solution:

Let
$$P(x, y) = xy^2$$
 and $Q(x, y) = 4x^2y$. By Green's Theorem

$$\oint_C xy^2 dx + 4x^2 y dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \int_0^2 \int_x^{2x} (8xy - 2xy) dy dx$$

$$= \int_0^2 \int_x^{2x} 6xy dy dx$$

$$= \int_0^2 3xy^2 \Big|_x^{2x} dx$$

$$= \int_0^2 9x^3 dx$$

$$= \frac{9}{4}x^4 \Big|_0^2$$

$$= 36.$$

