1. Be able to identify dependent and independent variables and order of a diff eqn
2. Be able to tell whether an ODE(ordinary differential equation) is separable, linear, non-linear, homogeneous, Bernoulli, exact or a combination of some
3. Be able to solve the above mentioned (#2) diff eqns
4. Be able to determine an integrating factor that makes an equation exact and solve the equation
5. Be able to use the above methods to solve problems related to falling body, mixing tank and Newton’s Law of cooling.

Review HWs, class notes, and the following questions

1. State the order of the ordinary differential equation, list the independent and dependent variables. Classify the equation as linear or nonlinear (state YES or NO), if the equation is nonlinear, circle the part of the equation that makes it so.

<table>
<thead>
<tr>
<th>ODE</th>
<th>order</th>
<th>dependent variable</th>
<th>independent variable</th>
<th>linear?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 \frac{dx}{dt} + \ln(t^2) x = \cos t$</td>
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<tr>
<td>$y^{(5)} + yy' = \tan x$</td>
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<tr>
<td>$(t - 3)y' + (\ln t)y = 2t$</td>
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<tr>
<td>$\frac{dy}{dt} + ty^2 = 0$</td>
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<tr>
<td>$y^{(4)} + t^2y' = e^t \sin t - y$</td>
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</tbody>
</table>
2. Find the general solution of the following linear ODEs (ordinary differential equations):

(a) \( x \frac{dy}{dx} + y = xe^x, \quad x > 0 \)

(b) \( \frac{dy}{dt} - y \tan t = \tan t, \quad -\pi/2 < t < \pi/2 \)

(c) \( \frac{dy}{dx} = \frac{y}{y \cos y - x}, \quad y > 0 \)

3. Solve the following ODE/IVP (initial value problem).

(a) \( (x^2 - 9) \frac{dy}{dx} + xy = 0 \)

(b) \( t \frac{dx}{dt} - 4x = t^6 e^t, \quad t > 0 \)

(c) \( \frac{dy}{dx} = x \sqrt{1 - y^2} \)
4. Solve the following separable ODEs.

(a) \( x \, dx + ye^{-x} \, dy = 0, \quad y(0) = 1 \)

(b) \( (y^2 + 1) \, dx = y \sec^2 x \, dy \)

(c) \( \frac{dy}{dx} = y^2 - 4 \)

5. Solve the following ODEs.

(a) (Homogeneous) \( (x^2 + y^2) \, dx + (x^2 - xy) \, dy = 0 \) (page 72 newbook)

(b) (Homogeneous) \( \frac{dy}{dx} = \frac{y - x}{y + x} \)

(c) \( xy^3 \frac{dy}{dx} = y^3 - x^3, \quad y(1) = 2 \)
6. Solve the ODE (Bernoulli): \[ x \frac{dy}{dx} + y = x^2 y^2 \]

7. Find the solution of the following initial value problems (IVPs):

   (a) \[ \frac{dy}{dt} - 2y = e^{2t}, \quad y(0) = 2 \]

   (b) \[ \frac{dy}{dx} + \frac{e^x}{e^x + 1} y = \frac{x}{e^x + 1}, \quad y(0) = 1 \]

   (c) \[ \cos(y) \frac{dx}{dy} + \sin(y)x = 2 \cos^3(y) \sin(y) - 1, \quad x \left( \frac{\pi}{4} \right) = 3\sqrt{2}, \quad 0 \leq y < \frac{\pi}{2} \]

   (d) \[ \frac{dy}{dx} = \frac{xy^3}{\sqrt{1 + x^2}}, \quad y(0) = -1 \]
(e) (Bernoulli) \( \frac{dy}{dt} - 2y = ty^4, \quad y(0) = 2 \)

8. Show that the following equation is exact, then find the general solution.

\[
(y/x + 6x)dx + (\ln x - 2)dy = 0, \quad x > 0
\]
9. Find the value $k$ so that the following equation is exact, then find the general solution.

$$(5x + ky) \, dx + (4x - 3y) \, dy = 0$$

10. Find an integrating factor for the following non-exact ode and use it to solve the ode.

$$2xy^3 \, dx + (3x^2y^2 + x^2y^3 + 1) \, dy = 0$$
11. Find an integrating factor for the following non-exact ode and use it to solve the ode.

\[(2x^2 + y) \, dx + (x^2 y - x)\, dy = 0\]

12. A 1000 gallon holding tank that catches runoff from some chemical process initially has 800 gallons of water with 100 ounces of pollution dissolved in it. Polluted water flows into the tank at a rate of 3 gal/hr and contains 5 ounces/gal of pollution in it. Simultaneously a well mixed solution leaves the tank at 2 gal/hr. Find the amount of pollution in the holding tank when it is full.
13. A tank contains 100 gal of pure water. A sugar-water solution containing 0.2 lb of sugar per gal enters the tank at a rate of 3 gal per minute and simultaneously a drain is opened at the bottom of the tank allowing the well stirred sugar solution to leave at 3 gal per minute.

(a) What will be the sugar content in the tank after 20 minutes?
(b) How long will it take the sugar content in the tank to reach 15 lb?
(c) What will be the eventual \( (t \to \infty) \) sugar content in the tank?

14. When a cake is removed from an oven, its temperature is measured at 300F. Three minutes later its temperature is 200F. How long will it take for the cake to cool off to 75F when it is placed in a room of temperature 70F?
15. Suppose a small cannonball weighing 5kg is shot vertically upward from a height of 200m from the ground with an initial velocity 10 m/s. Air resistance is $0.25|v|$. 

(a) Set up the governing IVP. This includes a differential equation and an initial condition.
(b) Find the velocity, $v(t)$ of the cannon as a function of time.
(c) Find the position, $x(t)$ of the cannon as a function of time.
(d) Find the velocity at which the ball lands.

16. A murder victim is discovered at midnight and the temperature of the body is recorded at $31^0 C$. One hour later, the temperature of the body is $29^0 C$. Assume that the surrounding air temperature remains constant at $21^0 C$. Calculate the victim’s time of death. Note: The normal temperature of a living human being is approximately $37^0 C$. 