1. State the order of the ordinary differential equation, list the independent and dependent variables. Classify the equation as linear or nonlinear (state YES or NO), if the equation is nonlinear, box the part of the equation that makes it so.

<table>
<thead>
<tr>
<th>ODE</th>
<th>order</th>
<th>dependent variable</th>
<th>independent variable</th>
<th>linear?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 \frac{dx}{dt} + \ln (t^2) x = \cos t$</td>
<td>1</td>
<td>$x$</td>
<td>$t$</td>
<td>No</td>
</tr>
<tr>
<td>$y^{(5)} + \frac{dy}{dx} y' = \tan x$</td>
<td>5</td>
<td>$y$</td>
<td>$x$</td>
<td>No</td>
</tr>
<tr>
<td>$(t - 3)y' + (\ln t)y = 2t$</td>
<td>1</td>
<td>$y$</td>
<td>$t$</td>
<td>yes</td>
</tr>
<tr>
<td>$\frac{dy}{dt} + t \frac{dy}{dt} = 0$</td>
<td>4</td>
<td>$y$</td>
<td>$t$</td>
<td>No</td>
</tr>
<tr>
<td>$y^{(4)} + t^2 y' = e^t \sin t - y$</td>
<td>4</td>
<td>$y$</td>
<td>$t$</td>
<td>Yes</td>
</tr>
</tbody>
</table>

2. Find the general solution of the following linear ordinary differential equations:

(a) $x \frac{dy}{dx} + y = xe^x, \quad x > 0 \quad$ Answer: $y(x) = e^x - \frac{e^x}{x} + \frac{C}{x}$.

(b) $\frac{dy}{dt} - y \tan t = \tan t, \quad -\pi/2 < t < \pi/2 \quad$ Answer: $(t) = C \sec t - 1$.

(c) $\frac{dy}{dx} = \frac{y}{y \cos y - x}, \quad y > 0$

Answer: $x(y) = \sin y + \frac{\cos y}{y} + \frac{C}{y}$. (The given ode is linear in $x$ w.r.t. $y$)

3. Solve the following ODEs.

(a) $(x^2 - 9) \frac{dy}{dx} + xy = 0, \quad$ Answer: $y(x) = \frac{C}{\sqrt{|x^2 - 9|}}$.

(b) $t \frac{dx}{dt} - 4x = t^6 e^t, \quad t > 0 \quad$ Answer: $x(t) = t^5 e^t - t^4 e^t + Ct^4$.

(c) $\frac{dy}{dx} = x \sqrt{1 - y^2} \quad$ Answer: $y(x) = \sin \left(\frac{1}{2} x^2 + C\right)$. 
4. Solve the following separable ODEs.

(a) \( x \, dx + ye^{-x} \, dy = 0, \quad y(0) = 1. \) Answer: \( y = \sqrt{2e^x(1 - x) - 1}. \)

(b) \( (y^2 + 1) \, dx = y \sec^2 x \, dy. \) Answer: \( \ln(y^2 + 1) = x + \frac{1}{2} \sin(2x) + C. \)

(c) \( \frac{dy}{dx} = y^2 - 4. \) Answer: \( \frac{1}{4} \ln \frac{|y - 4|}{|y + 2|} = x + C. \)

5. Solve the following ODEs.

(a) \( (x^2 + y^2) \, dx + (x^2 - xy) \, dy = 0 \) Ans: \( \frac{y}{x} - 2 \ln |1 + y/x| = \ln |x| + C. \)

(b) \( \frac{dy}{dx} = \frac{y - x}{y + x} \) Ans: \( \tan^{-1}(y/x) + \frac{1}{2} \ln(1 + y^2/x^2) = C - \ln |x|. \)

(c) \( xy^2 \frac{dy}{dx} = y^3 - x^3, \quad y(1) = 2. \) Ans: \( \frac{y^3}{3x^3} = \frac{8}{3} - \ln |x|. \)

6. Solve the ODE \( x \frac{dy}{dx} + y = x^2 y^2. \) Answer: \( y = \frac{1}{Cx - x^2}. \)

7. Find the solution of the following initial value problems (IVPs):

(a) \( \frac{dy}{dt} - 2y = e^{2t}, \quad y(0) = 2 \) Ans: \( y(t) = e^{2t}(t + 2). \)

(b) \( \frac{dy}{dx} + \frac{e^x}{e^x + 1} y = \frac{x}{e^x + 1}, \quad y(0) = 1 \) Ans: \( y(x) = \frac{x^2 + 4}{2(e^x + 1)}. \)

(c) \( \cos(y) \frac{dx}{dy} + \sin(y)x = 2 \cos^3(y) \sin(y) - 1, \quad x\left(\frac{\pi}{4}\right) = 3\sqrt{2}, \quad 0 \leq y < \frac{\pi}{2} \) Ans: \( y = -\frac{1}{2} \cos(y) \cos(2y) - \sin(y) + 7 \cos(y). \)

(d) \( \frac{dy}{dx} = \frac{xy^3}{\sqrt{1 + x^2}}, \quad y(0) = -1 \) Ans: \( y = -\frac{1}{\sqrt[3]{3 - 2\sqrt{1 + x^2}}}. \)

(e) \( 6 \frac{dy}{dt} - 2y = ty^4, \quad y(0) = 2 \) Ans: \( \frac{e^t}{y^3} = -\frac{1}{2} e^t(t - 1) - \frac{3}{8}. \)

8. Show that the following equation is exact, then find the general solution.

\( (y/x + 6x) \, dx + (\ln x - 2) \, dy = 0, \quad x > 0 \)

Ans: \( (\ln x)y + 3x^2 - 2y = C. \)

9. Find the value \( k \) so that the following equation is exact, then find the general solution.

\( (5x + ky) \, dx + (4x - 3y) \, dy = 0 \)

Ans: \( k = 4; \quad 5/2 x^2 + 4xy - 3/2 y^2 = C. \)
10. Find an integrating factor for the following non-exact ode and use it to solve the ode.

\[ 2xy^3 \, dx + (3x^2y^2 + x^2y^3 + 1) \, dy = 0 \]

Ans: \( e^y(x^2y^3 + 1) = C \)

11. Find an integrating factor for the following non-exact ode and use it to solve the ode.

\[ (2x^2 + y) \, dx + (x^2y - x) \, dy = 0 \]

Ans: \( 2x - \frac{y}{x} + \frac{y^2}{2} = C \)

12. A 1000 gallon holding tank that catches runoff from some chemical process initially has 800 gallons of water with 100 ounces of pollution dissolved in it. Polluted water flows into the tank at a rate of 3 gal/hr and contains 5 ounces/gal of pollution in it. Simultaneously a well mixed solution leaves the tank at 2 gal/hr. Find the amount of pollution in the holding tank when it is full. Ans: 2504 oz

13. A tank contains 100 gal of pure water. A sugar-water solution containing 0.2 lb of sugar per gal enters the tank at a rate of 3 gal per minute and simultaneously a drain is opened at the bottom of the tank allowing the well stirred sugar solution to leave at 3 gal per minute.

(a) What will be the sugar content in the tank in the tank after 20 minutes?
(b) How long will it take the sugar content in the tank to reach 15 lb?
(c) What will be the eventual \((t \to \infty)\) sugar content in the tank?


14. When a cake is removed from an oven, its temperature is measured at 300°F. Three minutes later its temperature is 200°F. How long will it take for the cake to cool off to 75°F when it is placed in a room of temperature 70°F? Ans: 20.1 min

15. Suppose a small cannonball weighing 10kg is shot vertically upward from a height of 200m from the ground with an initial velocity 100 m/s. Air resistance is 0.25|v|.

(a) Set up the governing IVP. This includes a differential equation and an initial condition.
(b) Find the velocity, \(v(t)\) of the cannon as a function of time.
(c) Find the position, \(x(t)\) of the cannon as a function of time.
(d) Find the velocity at which the ball lands.

16. A murder victim is discovered at midnight and the temperature of the body is recorded at 31°F. One hour later, the temperature of the body is 29°F. Assume that the surrounding air temperature remains constant at 21°F. Calculate the victim’s time of death. Note: The normal temperature of a living human being is approximately 37°F. Ans: 9:54PM