Homework 1

1. Determine if each of the following equations is an ordinary differential equation or a partial differential equation and state the independent and dependent variables. If the equation is an ordinary differential equation then determine (a) the order of the ODE; and (b) if the equation is linear or nonlinear.

(a) \[ \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x^3 \]
(b) \[ \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \]
(c) \[ (2x - y)dx - ydy = 0 \]

2. Verify that the function

\[ \phi(x) = c_1 e^{-x} + c_2 e^{2x} \]

is an explicit solution to the given differential equation

\[ y'' - y' - 2y = 0 \]

for any choice of constants \( c_1 \) and \( c_2 \).

3. Determine whether or not the relation \( x^2 + y^2 = 4 \) is an implicit solution to the following differential equations.

(a) \[ \frac{dy}{dx} = \frac{x}{y} \]
(b) \[ \frac{dy}{dx} = \frac{x}{y} \]

4. Determine whether or not the relation \( \sin(xy) = y + x \) is an implicit solution to the differential equation

\[ \frac{dy}{dx} = \frac{1 - y \cos(xy)}{\cos(xy) - 1} \]

Text book Pg. 14 #2e, #3 (Find all the equation solutions.) c, f, #4e, #6a