How to Calculate a Two-Factor ANOVA
Balanced (any) or Unbalanced (2x2 only)
Note

• This is not the most efficient way to do the calculations.

• It is not the method you will find in most textbooks.

• But it allows you to see what the calculations are actually doing.

• If the design is unbalanced, it gives what are called Type III sums of squares (for the 2x2 case only).
Get sum ($\sum X$), sumsq ($\sum X^2$), n, and mean in all cells

• add up all the scores in a cell to get sum
• square the scores and add up the squared values to get sumsq
• count ‘em up to get n
• in most problems, I’ve given you these values
• when you calculate the means, carry enough precision (decimal places) to make your final answers accurate - 3 or 4 decimal places at least
Calculate the sum of squares in each cell

- use any of the following
  - $SS = \text{sumsq} - \text{sum}^2 / n$
  - $SS = \sum X^2 - (\sum X)^2 / n$
  - $SS = \sum (X - M)^2$
Error term
(unexplained variability)

- add up the sums of squares in the cells (any variability inside a cell is unexplained)

- $SS_W = \text{sum}(SSs \text{ in the cells})$

- $df_W = N - k$ (total no. of subjects minus number of groups)

- $MS_W = \frac{SS_W}{df_W}$
  - $MS_W$ is the pooled variance
  - $\sqrt{MS_W} = \text{pooled standard deviation}$
Total variability

- add across the rows and down the columns to get marginal sum, sumsq, and n
- add across the column marginals OR down the row marginals to get grand sum, sumsq, and n for the entire table
- using these grand totals (i.e., treating all scores as if they were from one large group)
  - $\text{SST} = \text{sumsq} - \frac{\text{sum}^2}{\text{n}}$ (where $\text{n} = \text{N}$)
  - $\text{df}_T = \text{N} - 1$
  - MST is not calculated
- if the design is unbalanced, it is not necessary to calculate SST unless you want effect sizes
Explained variability

- if balanced
  - $SS_{Between} = SST - SSW$

- if unbalanced or balanced ("means method") - cell means varying around the GM
  - calculate the grand mean:
    $$GM = \frac{\text{sum(cell means)}}{\text{no. of cells}} = \frac{\text{grand sum}}{N} \quad \text{(if balanced!)}$$

  - calculate the effective group size:
    $$ne = \frac{k}{\text{sum}(1/n)} \text{ where } k \text{ is no. of groups and } n \text{ is no. of subjects cell by cell}$$

  - $SS_{Between} = ne \times \text{sum(cell mean - GM)}^2$
Partitioning SSBetw

- In the balanced case, SSA and SSB can be calculated as if you were doing a oneway ANOVA. This doesn’t work in the unbalanced case with Type III sums of squares (although it seems like it ought to).

- calculate the row and column marginal means
  - rows: add the means across columns and divide by the number of columns
  - columns: add the means down rows and divide by the number of rows

- these are the unweighted marginal means!
Partitioning SSBetw

- row effect (SSA) - main effect of A (row means varying around the GM)

- get ne in the rows
  \[ ne_r = \text{ne per cell} \times \text{no. of columns} \]

- SSA = \( ne_r \times \sum(\text{row mean} - \text{GM})^2 \)

- dfA = no. of rows - 1

- MSA = SSA / dfA
Partitioning SSBetw

- column effect (SSB) - main effect of B (column means varying around the GM)
  - get ne in the columns
    \[ ne_c = \text{ne per cell} \times \text{no. of rows} \]
  - \[ \text{SSB} = ne_c \times \text{sum(column mean - GM)}^2 \]
- \[ \text{dfB} = \text{no. of columns} - 1 \]
- \[ \text{MSB} = \text{SSB} / \text{dfB} \]
Partitioning SSBetw

- interaction effect (SSAxB) - what’s left over
  - SSAxB = SSBetw - SSA - SSB
  - dfAxB = dfT - dfW - dfA - dfB
    = dfA * dfB
  - MSAxB = SSAxB / dfAxB

- F-ratios
  - as always: F (for any effect) = MSeffect / MSW
  - get F-ratios for all three effects