Factorial ANOVA
(Balanced Designs Only!)
The complete oddmonkeys data

```
# oddmonkeys.txt
# Data are from monkeys performing an oddity task in which they are
# exposed to three objects, two of which are the same. Their task is
# to learn that reward is hidden under the odd object. Reward consists
# of one grape (low), three grapes (moderate), or five grapes (high).
# Monkeys were randomly assigned to these conditions. In addition, the
# motivation of the monkeys to learn the task was manipulated by food
# depriving them for either one hour before doing the task (weak) or
# 24 hrs (strong). Monkeys were also randomly assigned to these conditions.
# A total of 30 learning trials were administered. The DV, successes, is
# the number of times out of the 30 that a monkey made the correct choice.
#
<table>
<thead>
<tr>
<th>successes</th>
<th>reward</th>
<th>motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>low</td>
<td>weak</td>
</tr>
<tr>
<td>4</td>
<td>low</td>
<td>weak</td>
</tr>
<tr>
<td>0</td>
<td>low</td>
<td>weak</td>
</tr>
<tr>
<td>7</td>
<td>low</td>
<td>weak</td>
</tr>
<tr>
<td>15</td>
<td>low</td>
<td>strong</td>
</tr>
<tr>
<td>6</td>
<td>low</td>
<td>strong</td>
</tr>
<tr>
<td>10</td>
<td>low</td>
<td>strong</td>
</tr>
<tr>
<td>13</td>
<td>low</td>
<td>strong</td>
</tr>
<tr>
<td>13</td>
<td>moderate</td>
<td>weak</td>
</tr>
<tr>
<td>5</td>
<td>moderate</td>
<td>weak</td>
</tr>
<tr>
<td>7</td>
<td>moderate</td>
<td>weak</td>
</tr>
<tr>
<td>7</td>
<td>moderate</td>
<td>weak</td>
</tr>
<tr>
<td>15</td>
<td>moderate</td>
<td>weak</td>
</tr>
<tr>
<td>6</td>
<td>moderate</td>
<td>strong</td>
</tr>
<tr>
<td>18</td>
<td>moderate</td>
<td>strong</td>
</tr>
<tr>
<td>9</td>
<td>moderate</td>
<td>strong</td>
</tr>
<tr>
<td>14</td>
<td>moderate</td>
<td>strong</td>
</tr>
<tr>
<td>6</td>
<td>high</td>
<td>weak</td>
</tr>
<tr>
<td>8</td>
<td>high</td>
<td>weak</td>
</tr>
<tr>
<td>6</td>
<td>high</td>
<td>weak</td>
</tr>
<tr>
<td>13</td>
<td>high</td>
<td>weak</td>
</tr>
<tr>
<td>14</td>
<td>high</td>
<td>strong</td>
</tr>
<tr>
<td>17</td>
<td>high</td>
<td>strong</td>
</tr>
<tr>
<td>13</td>
<td>high</td>
<td>strong</td>
</tr>
</tbody>
</table>
```

Notice there are two IVs, reward and motivation...

...and that they are crossed with each other.

Meaning every level of reward goes with every level of motivation: low with weak, low with strong, moderate with weak, etc.
Factorial Designs

- There are multiple categorical IVs.
- These are crossed with each other (ideally, completely crossed, meaning every possible combination of the IVs occurs.)
- All of the groups this creates should be independent (in the simplest cases).
- Although it’s not strictly necessary, ideally, there will be “replication” - which means more than one observation in each of those groups.
- There should be a numeric DV.
Factorial Design Speak

• Remember your vocabulary. It is especially important when talking about factorial designs.

• An IV is also referred to as a “factor.”

• Individual values within a factor are called “levels.”

• Example: reward is a factor. Low, moderate, and high are levels of reward.
First Question!

Is this a balanced design?

You cannot tell from the summary if the design is balanced!
You have to look at all the individual groups!

8 subjects in each level of reward, 12 subjects in each level of motivation.
A Moment of Truth

- Balanced designs are not difficult to calculate by hand.
- But they are tedious and it’s easy to lose your way in the midst of the calculations.
- Unbalanced factorial designs are extremely difficult to calculate by hand. (We’re talking DAYS of calculations in some cases!)
- Therefore, R to the rescue.
Is There an Alternative to R?

- You can learn to do balanced factorial ANOVAs by hand. There are plenty of resources online that will show you how.
- You can learn to do it in SPSS. I haven’t used SPSS in over 20 years, so you’re on your own with that.
- Excel can also do balanced factorial ANOVAs. I don’t even have Excel on my computer.
- There is also Vassarstats.net, but that won’t work for unbalanced designs.
What if it’s not balanced?

• Then you have serious issues to deal with.

• You’ll have to decide between weighted vs. unweighted means analysis (not covered yet).

• This was a fairly minor issue in single-factor ANOVA.

• It is now a MAJOR issue!

• The good news is, if the design is balanced, they both give the same answer, so there is no choice to be made.

• ALWAYS CHECK for a balanced design!
Other Problems Created by Unbalanced Designs

- There is a loss of power, which means the ANOVA is less likely to find an effect if one exists.
- The ANOVA becomes more sensitive to violation of its assumptions, which have to be checked more carefully.
- This was also true of the t-test and single-factor ANOVA, but it’s really, really true for factorial and repeated measures designs.
There are assumptions, but they should sound very familiar to you.

• random sampling (or random assignment)
• independence of cases (i.e., not paired, not matched, no subject can influence what any other subject does)
• normal distributions
• homogeneity of variance
Factorial designs can become incredibly complex!

- We’ll stick to the simplest ones (for now).
The ANOVA “design table”

<table>
<thead>
<tr>
<th></th>
<th>low</th>
<th>moderate</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>weak motivation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>strong motivation</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is called a 2x3 factorial design.

So how many groups are there? There are 2x3 = 6 groups, and there should be data in all of them.
What effects can we see?

• We can see an effect of each IV.

• These are called main effects.
  • main effect of motivation
  • main effect of reward strength

• We can also see an effect of both IVs acting together.

• This is called an interaction.
  • motivation X reward strength interaction “by”
* main effects are seen in the marginal means
* to see interactions, we have to examine the cell means

<table>
<thead>
<tr>
<th></th>
<th>low</th>
<th>moderate</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>weak</td>
<td>3.0</td>
<td>10.0</td>
<td>14.0</td>
</tr>
<tr>
<td>strong</td>
<td>11.0</td>
<td>12.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>

cell (or group) mean: 9.0
row marginal means: 11.0
grand mean: 11.0
column marginal means: 7.0 11.0 12.0 10.0
Simple Effects

* simple effects are the effect of one variable at one level of the other variable
* every row and every column in the design table is a simple effect
* when simple effects differ across levels of the other variable, that’s an interaction

**reward magnitude**

<table>
<thead>
<tr>
<th></th>
<th>low</th>
<th>moderate</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>weak</td>
<td>3.0</td>
<td>10.0</td>
<td>14.0</td>
</tr>
<tr>
<td>strong</td>
<td>11.0</td>
<td>12.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>

* this is a simple effect (of motivation at low reward)
* a simple effect is one row or one column of the table
Graphing: Interaction Plots (a.k.a., Profile Plots)

The levels of one variable are plotted along the x-axis. The levels of the other variable are plotted as separate lines on the graph. The DV is on the y-axis (as always). Cell means are plotted on the graph.
The ANOVA

As usual, SS.total is partitioned into explained and unexplained variability, but then the explained variability has to be further partitioned into the individual effects. A significance test is calculated on each of the effects.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>motivation</td>
<td>1</td>
<td>24</td>
<td>24</td>
<td>1.309</td>
<td>0.27</td>
</tr>
<tr>
<td>reward</td>
<td>2</td>
<td>112</td>
<td>56</td>
<td>3.055</td>
<td>0.07</td>
</tr>
<tr>
<td>motivation x reward</td>
<td>2</td>
<td>144</td>
<td>72</td>
<td>3.927</td>
<td>0.04</td>
</tr>
<tr>
<td>error (within)</td>
<td>18</td>
<td>330</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>23</td>
<td>610</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In R...

```r
> aov.out = aov(successes ~ motivation * reward, data=OM)
> summary(aov.out)

                 Df Sum Sq Mean Sq   F value Pr(>F)
motivation       1   24  24.000  1.309000 0.2675
reward           2 112  56.000  3.054968 0.0721 .
motivation:reward 2 144  72.000  3.926875 0.0384 *
Residuals       18 330  18.333
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

> 24 + 112 + 144 + 330  # SS.total
[1] 610
```

Neither main effect was significant. There was a significant motivation-by-reward interaction.

Effect sizes (eta-squared) = SS.effect / SS.total
* motivation: 24 / 610 = .04 (small)
* reward: 112 / 610 = .18 (moderate)
* interaction: 144 / 610 = .24 (moderate)

All three effects are tested, giving a p-value for each one.
Describing the Interaction

* Here it is graphed a little differently.
* I would say, “When motivation was weak, magnitude of reward had a substantial effect on learning, but when motivation was strong, magnitude of reward had very little effect.”
* Describe an interaction by describing how the simple effects change.