The Chi-Square Tests

The statistical procedures we have considered so far are **parametric** statistical tests. Parametric tests, such as z, t, and F, all make assumptions about the shape of the population distribution and require use of data which represent an interval or ratio scale of measurement (for example, temperature, height, weight, scores on a standardized test, etc.). Interval and ratio measures indicate <u>how much</u> of a characteristic the subjects exhibit. **Nonparametric** statistical tests are available for analyzing data that do not reflect interval or ratio measures. There are many nonparametric procedures available to analyze ordinal (rank order) data and nominal (categorical) data. These procedures do not generally make stringent assumptions about the shapes of distributions. The chi-square tests are among the nonparametric procedures.

The chi-square statistic is described here. This test is applied to data representing the nominal scale of measurement, often called **frequency data**. Nominal measurement involves the assignment of subjects to one or more discrete categories and provides information regarding the frequency of occurrence within each category. From nominal data we get <u>no</u> information about how much of a characteristic exists, only information as to whether or not that individual has the trait at all. For example, we can construct a nominal scale for political party affiliation by grouping individuals according to party loyalties: so many Democrats, so many Republicans, so many Independents, etc. We cannot determine the strength or amount of any person's party affiliation from this nominal scale, we can only know what that person's party choice is. In short, nominal data are generated by sorting and counting: sorting the data into discrete, mutually exclusive categories and then counting the frequency of occurrence within each category.

The Chi-Square Test for Goodness of Fit

This test is used to evaluate how well the frequency distribution for a sample fits a hypothesized distribution proposed by the null hypothesis.

A) Testing the hypothesis of no preference or equally distributed frequencies

Suppose a marketing researcher wishes to examine student preferences for four brands of personal computers: Compaq, Dell, Gateway, and IBM. The researcher sets alpha at .01 and tests the null hypothesis that preferences are equally distributed across the four brands. The researcher selects a random sample of 100 students, and each student is asked which of the four brands he or she prefers. The observed frequencies are as follows: 20 students prefer Compaq, 30 prefer Dell, 40 prefer Gateway, and 10 prefer IBM.

1) State the null and alternative hypotheses.

H_0: Student preferences are equally divided among the four computer brands.

H_1: Student preferences are not equally divided among the four computer brands.

2) Establish a decision criterion.

Note: degrees of freedom for the goodness of fit test is df = C - 1, where C is the number of categories the subjects have been classified into. The critical value of chi-square can be obtained from the table in the back of the book.

3) Collect the data and do the analysis.

	O	**E**	**(O - E)**	**(O - E)^2**	**(O - E)^2/E**
Compaq	20				
Dell	30				
Gateway	40				
IBM	10				

Note: $\chi^2 = \sum \frac{(O - E)^2}{E}$

4) Make a statistical decision concerning the null hypothesis.

5) Write a conclusion.

The students showed a significant computer brand preference, $\chi^2(3, n = 100) = 20.0, p < .01$. Among this sample of students, Gateway was the most preferred brand, and IBM was the least preferred brand.

B) Testing the hypothesis of no difference from a comparison population

The chi-square goodness of fit test can also be used to compare an observed frequency distribution to a frequency distribution that is known to exist for a specific population. Suppose that a 1990 survey conducted in South Carolina revealed that 55% of residents favored increasing the tax on cigarettes while 45% opposed a tax increase. In 2000, the survey was conducted again using a random sample of $n = 500$ residents. Of these individuals, 330 favored the cigarette tax increase, and 170 opposed the tax increase.

1) State the null and alternative hypotheses.

H_0: There is no change in the distribution of opinions: 55% in favor, 45% opposed.

H_1: There has been a change in the distribution of opinions since the 1990 survey.

2) Establish a decision criterion.

3) Collect the data and do the analysis.

	O	**E**	**(O - E)**	**$(O - E)^2$**	**$(O - E)^2/E$**
Favor tax	330				
Oppose tax	170				

4) Make a statistical decision concerning the null hypothesis.

5) Write a conclusion.

There has been a significant change in the distribution of opinions concerning the cigarette tax, $\chi^2(1, n = 500) = 24.44$, $p < .05$. The results of the 2000 survey indicated that 66% of the residents favored the cigarette tax increase, and 34% opposed the tax increase.

The Chi-Square Test of Independence

Chi square can also be used to determine if there is a relationship between two nominal or categorical variables. In this case, each subject in a sample is categorized on two nominal variables. The resulting table is called a **contingency table** or a **cross-tabulation**.

Can you increase the response rate for a mail survey by prenotifying the respondents that a survey will be mailed to them? A study by Stafford (1966) addressed this question by comparing the survey response rate of three groups of subjects. The first group of subjects received a preliminary letter describing the survey that would subsequently be mailed to them. The second group of subjects received a phone call describing the survey that would subsequently be mailed to them. The third group of subjects received no prenotification.

1) State the null and alternative hypotheses.

H_0: There is no relationship between method of prenotification and rate of survey return.

H_1: There is a relationship between method of prenotification and rate of survey return.

2) Establish a decision criterion.

3) Collect the data and do the analysis.

Observed frequencies

	Letter	**Phone call**	**None**
Returned survey	171	146	118
Did not return survey	220	68	455

Note: expected frequency for a cell = (row total)(column total) / N

Expected frequencies

	Letter	**Phone call**	**None**
Returned survey			
Did not return survey			

Note: chi-square values for each cell are calculated as above, $(O - E)^2/E$.

Chi-square values

	Letter	**Phone call**	**None**
Returned survey			
Did not return survey			

4) Make a statistical decision concerning the null hypothesis.

5) Write a conclusion.

There was a significant relationship between prenotification strategy and response to the survey, $\chi^2(2, n = 1178) = 163.43$, $p < .01$. The phone call was the most effective strategy, producing a survey return rate of 68%. The return rate for the letter strategy was 44% and the return rate with no prenotification was 21%.

Assumptions and Restrictions on the Use of Chi-Square Tests

1. The chi square test can be used only with frequency data resulting from categorizing subjects.
2. The sample should be randomly sampled from a population to which inference is to be made.
3. Each subject can contribute only once to the frequencies in the table. A subject's response cannot be classified into more than one category.
4. The sum of the observed frequencies and the sum of the expected frequencies must be equal.
5. The chi-square tests are approximations to exact techniques which are very tedious to calculate. To make these approximates reasonably accurate, the *expected frequencies* in all cells should be at least 5.