$\qquad$

1. (18 pts) Evaluate the following limits. Show work.
(a) $\lim _{x \rightarrow 0} \frac{1-\cos (x)}{x^{2}+x}=$
(b) $\lim _{x \rightarrow \infty} \frac{(\ln x)^{2}}{x}=$
(c) $\lim _{x \rightarrow-2} \frac{x+2}{x^{2}+3 x+2}=$
2. ( 15 pts ) Find the absolute maximum and absolute minimum values of the function

$$
f(x)=10+27 x-x^{3}, \quad \text { on the interval }[0,4] .
$$

Show your work.
absolute maximum: $\qquad$ absolute minimum:
3. ( 15 pts ) The figure below shows the graph of the derivative $\mathbf{f}^{\prime}$ of a function $f$.

(a) On what intervals is $f$ increasing or decreasing? Explain.
(b) For what $x$-values does $f$ have a local maximum or minimum? Explain.
(c) For what $x$-values does $f$ have a point of inflection? Explain.
4. ( 15 pts ) Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.
(a) If $f^{\prime}(c)=0$, then $f$ has a local maximum or minimum at $c$.
(b) If $f$ has an absolute minimum at $c$, then $f^{\prime}(c)=0$.
(c) If $f$ is continuous on the open interval $(a, b)$, then $f$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some numbers $c$ and $d$ in $(a, b)$.
(d) If $f$ is differentiable and $f(-1)=f(1)$, then there is a number $c$ such that $-1<c<1$ and $f^{\prime}(c)=0$.
(e) By l'Hospital's rule, $\lim _{x \rightarrow \infty} x^{2} e^{-x}=\lim _{x \rightarrow \infty}-2 x e^{-x}$.
5. (12 pts)
(a) Recall that the Mean Value Theorem states that if $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then

$$
f^{\prime}(c)=\frac{f(b)-f(a)}{b-a}
$$

for some real number $c$ in the interval $(a, b)$.
Draw the graph of a function over the interval $[0,5]$ which satisfies the hypotheses of the Mean Value Theorem as well as the corresponding secant and tangent lines which satisfy the conclusion. Make sure to label your graph with as much information as possible.

(b) The function $f(x)=3 x^{2}+2 x+5$ is continuous on $[-1,1]$ and differentiable on $(-1,1)$. Find all numbers $c$ in $(-1,1)$ satisfying the conclusion of the Mean Value Theorem. Show your work.
6. (10 pts) Sketch the graph of a function that satisfies the given conditions:
(a) $f(0)=-1$ and $f(3)=2$,
(b) $\lim _{x \rightarrow-2} f(x)=-\infty, \lim _{x \rightarrow 1^{-}} f(x)=-\infty$, and $\lim _{x \rightarrow 1^{+}} f(x)=\infty$,
(c) $\lim _{x \rightarrow-\infty} f(x)=1$,
(d) $f^{\prime}(x)>0$ on the interval $(-2,0)$ and $(3, \infty)$,
(e) $f^{\prime}(x)<0$ on the intervals $(-\infty,-2),(0,1)$, and $(1,3)$
(f) $f^{\prime \prime}(x)>0$ on the intervals $(1, \infty)$
(g) $f^{\prime \prime}(x)<0$ on the intervals $(-\infty,-2)$ and $(-2,1)$.

7. (15 pts) Choose one of the following problems. Circle the one you want to have graded. You must show your work in order to receive full credit.
(a) At which points on the curve $y=1+40 x^{3}-3 x^{5}$ does the tangent line have the largest slope?
(b) A farmer wants to fence a rectanglular field with an area of 1.5 million square feet and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

