MATH 160 Exam 3 **Name** ______________________________

CCU Dept. of Math/Stats

Sample A **Score** ________

1. (18 pts) Evaluate the following limits. Show work.

   (a) $\lim\limits_{x \to 1} \dfrac{x^9 - 1}{x^5 - 1} =$

   (b) $\lim\limits_{x \to \infty} \dfrac{x^2}{\mathrm{e}^x} =$

   (c) $\lim\limits_{x \to 0^+} x\ln(x) =$

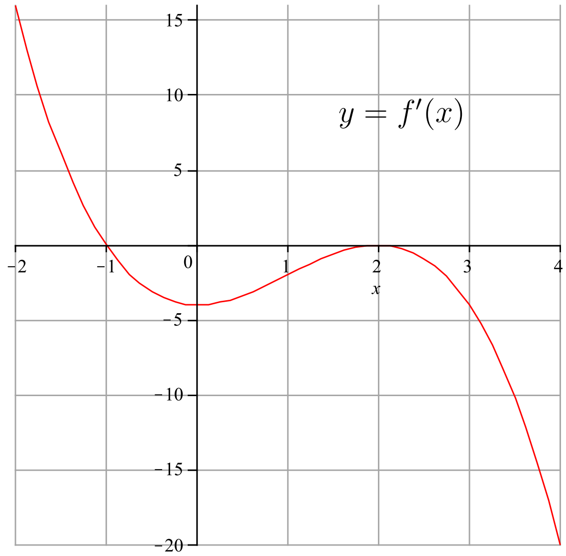
2. (13 pts) Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^3 - 6x^2 + 9x + 2 \quad \text{on the interval } [2,\, 4].$$

   Show your work.

   absolute maximum: ____________ absolute minimum: ____________

3. (15 pts) The figure below shows the graph of the **derivative** f' of a function f.



(a) On what intervals is f increasing or decreasing? Explain.

(b) For what x-values does f have a local maximum or minimum? Explain.

(c) For what x-values does f have a point of inflection? Explain.

4. (16 pts) Below is a function f and its first and second derivatives.

$$f(x) = \frac{2x^2}{x^2-1}, \quad f'(x) = \frac{-4x}{(x^2-1)^2}, \quad \text{and} \quad f''(x) = \frac{12x^2+4}{(x^2-1)^3}.$$

Use these functions to answer the following questions.

(a) Determine any horizontal and vertical asymptotes of f.

(b) Determine the critical number(s) of f.

(c) Determine the intervals on which the graph of $y = f(x)$ is increasing or decreasing. Determine local maxima or local minima of f, if any.

(d) Determine the intervals on which the graph of $y = f(x)$ is concave upward or concave downward. Determine inflection points of f, if any.

5. (12 pts) Find the antiderivative F of $f(x) = 5x^4 - 12x^5$ that satisfies $F(1) = 3$.

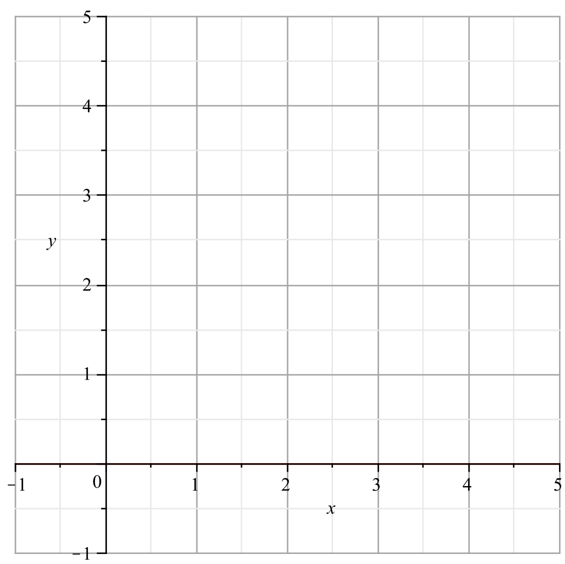
6. (12 pts)

(a) Recall that the Mean Value Theorem states that if f is continuous on $[a, b]$ and differentiable on (a, b), then

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

for some real number c in the interval (a, b).

Draw the graph of a function over the interval $[1, 4]$ which satisfies the hypotheses of the Mean Value Theorem as well as the corresponding secant and tangent lines which satisfy the conclusion. Make sure to label your graph with as much information as possible.



(b) The function $f(x) = 3x^2+2x+5$ is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$. Find all numbers c in $(-1, 1)$ satisfying the conclusion of the Mean Value Theorem. Show your work.

7. (14 pts) Choose one of the following problems. **Circle** the one you want to have graded. You must show your work in order to receive full credit.

   (a) A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. Find the dimensions of the rectangle that yield the largest possible total area of the four pens.

   (b) If 1200cm^2 of material is available to make a box with a square base (THIS BOX HAS A TOP). Find the dimensions that maximize the volume of the box.