MATH 160 Exam 2
CCU Dept. of Math/Stats
October 9, 2009

Name $\qquad$
Score $\qquad$

1. (a) (12 points) Use implicit differentiation to find $\frac{d y}{d x}$.

$$
y^{2}=x^{3}+3 x^{2}-y
$$

(b) (8 points) Find an equation of the line tangent to the graph of

$$
y^{2}=x^{3}+3 x^{2}-y
$$

at the point $(2,4)$.
2. (12 points) Compute $\frac{d y}{d x}$ for the following function. (Show work)

$$
y=x^{\sin x}
$$

3. Linear approximations.
(a) (6 points) Given a function $y=f(x)$ and a point ( $a, f(a)$ ) write the equation of the linearization of $f$ at $a$.

$$
L(x)=
$$

(b) (10 points) Use a linear approximation to estimate $\mathrm{e}^{0.09}$. (Show work).
4. (20 points) A rock is dropped from a height of 576 feet and falls toward earth in a straight line. In $t$ seconds the rock drops a distance of $s(t)=16 t^{2}$.
(a) How many seconds after release does the rock hit the ground?
(b) What is the velocity of the rock at $t$ seconds?
(c) How far has the rock fallen when its velocity is 96 feet per second?
(d) What is the instantaneous velocity of the rock when it hits the ground?
5. (16 points) Refer to the following table of values for the functions $f$ and $g$ and their derivatives to answer the following questions (Show work):

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | 7 | 4 | 1 |
| $g(x)$ | 3 | 0 | 5 |
| $f^{\prime}(x)$ | 2 | -3 | 1.5 |
| $g^{\prime}(x)$ | -1 | 6 | -2 |

(a) If $F(x)=f(x)+3 g(x)$, then $F^{\prime}(0)=$
(b) If $G(x)=\frac{g(x)}{f(x)}$, then $G^{\prime}(0)=$
(c) If $H(x)=\mathrm{e}^{g(x)}$, then $H^{\prime}(1)=$
(d) If $J(x)=g(f(x))$, then $J^{\prime}(2)=$
6. (16 points) Choose one of the following two related rates problems. You must show all of your work to receive full credit. If you attempt both, circle the one that you want to have graded.
(a) A baker forms a piece of dough into a cylinder. As he rolls it, the length, $L$ of the cylinder increases and the radius, $r$ decreases. If the length of the cylinder is increasing at 1.5 cm per minute, find the rate at which the radius is changing when the radius is 2.5 cm and the length is 60 cm . (Hint: the volume of a cylinder is $V=L \pi r^{2}$, and here $V$ is constant.)
(b) A balloon is rising vertically above a level, straight road at a constant rate of $2 \mathrm{ft} / \mathrm{s}$ (feet per second). At the moment when the balloon is 65 feet above the ground, a bicycle moving at a constant rate of $17 \mathrm{ft} / \mathrm{s}$ passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?

