

Name: _____

Instructions: Answer each of the following questions, and make sure you SHOW ALL YOUR WORK! Answers without supporting work will be counted as incorrect! When asked to explain your answers, use complete English sentences.

1. (20 pts) Find the following limits exactly. Approximate answers will not be accepted.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 2x}{x + 1}$

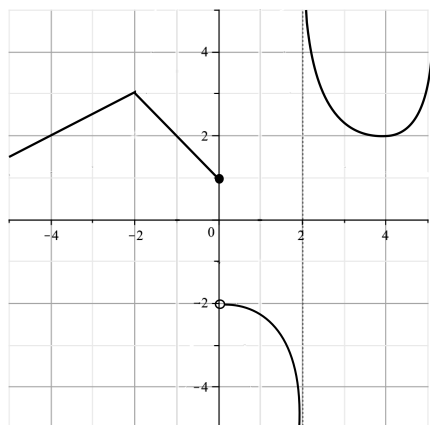
(b) $\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1}$

(c) $\lim_{x \rightarrow -2} \frac{1}{x + 2}$

(d) $\lim_{h \rightarrow 0} \frac{(1 + h)^2 - 1}{h}$

(e) $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x^2 - 1}{7x^3 - 4x + 2}$

For all questions on this page, consider the function $f(x)$ graphed below



2. (8 pts) Find the following values, if they exist.

(a) $\lim_{x \rightarrow 0^+} f(x) =$

(e) $f(0) =$

(b) $\lim_{x \rightarrow 0} f(x) =$

(f) $f(-2) =$

(c) $\lim_{x \rightarrow -2} f(x) =$

(g) $f'(-2) =$

(d) $\lim_{x \rightarrow 2^-} f(x) =$

(h) $f'(4) =$

3. (6 pts) At which points is $f(x)$ NOT continuous? For each, say why.

4. (6 pts) At which points is $f(x)$ NOT differentiable? For each, say why.

5. (6 pts) Sketch a graph of the derivative function $f'(x)$.

6. (18 pts) Kyle has modified his Roomba-brand robotic vacuum cleaner to travel in a straight line at high speeds (for racing purposes). The distance traveled by the bot after t minutes is given by $f(t)$ (in feet). Some values of the function $f(t)$ are given in the table below.

t	0	1	2	3	4
$f(t)$	0	15	40	100	220

- (a) What was the robot's average speed over the first four minutes of travel?
- (b) Approximate the robot's instantaneous velocity at $t = 3$.
- (c) Suppose the robot was traveling at exactly 45 feet per minute when $t = 2$. Find the equation of the tangent line to $f(t)$ at $t = 2$.

7. (10 pts) Let $f(x) = 5x + 4$. Use the definition of the derivative to find $f'(3)$.

8. (10 pts) Let $f(x) = \sqrt{2x + 1}$. Use the definition of the derivative to find $f'(x)$.

9. (16 pts) **True or False and explain:** Determine whether each statement below is true or false. If the statement is true, briefly explain why. If the statement is false, provide a counter-example.

(a) For the function $f(x) = e^x + \sin(\sqrt{x})$, we have $\lim_{x \rightarrow 2} f(x) = e^2 + \sin(\sqrt{2})$.

(b) If both the left-side limit and right-hand limit of a function exist at a point, then the function is continuous at that point.

(c) If a function is continuous at $x = 0$, then $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$ exists.

(d) By the Intermediate Value Theorem, every continuous function intersects the x -axis at some point.

10. **BONUS:** (10 pts) Find the value of k such that the function below is continuous. Show your work.

$$f(x) = \begin{cases} 4 - x^2 & x < 2 \\ 2x + k & x \geq 2 \end{cases}$$