MATH 160 Exam 1 CCU Dept. of Math/Stats Sample A

Name $\qquad$
Score $\qquad$

Instructions: Complete the following problems. Show work or give an explanation when told to do so in the problem. Please clearly indicate your final answer by boxing or underlining it. When applicable, include the units of your answer.

For problems 1-3, consider the function $f$ sketched below.


1. (12 points) Find the following values, if they exist.
(a) $\lim _{x \rightarrow-2^{-}} f(x)=$
(e) $f(1)=$ $\qquad$
(b) $\lim _{x \rightarrow-\infty} f(x)=$
(f) $f(-2)=$
(c) $\lim _{x \rightarrow 1^{+}} f(x)=$ $\qquad$
(g) $f^{\prime}(-1)=$ $\qquad$
(d) $\lim _{x \rightarrow 1} f(x)=$ $\qquad$
(h) $f^{\prime}(2)=$ $\qquad$
2. (10 points) Is $f$ continuous for the following values of $x$ ? In the table below, circle YES or NO. Explain your choice using the definition of continuity at a point.

| $x$ value | Continuous? | Reason |
| :---: | :---: | :--- |
| -2 | YES NO |  |
| 1 | YES NO |  |

3. (10 points) Does $f$ have a derivative at each of the following $x$ values? In the table below, circle YES or NO. Explain your choice using calculus.

| $x$ value | Differentiable? | Reason |
| :---: | :---: | :--- |
| 1 | YES NO |  |
| 3 | YES NO |  |

4. (24 points) Evaluate the following limits, if they exist. Use exact values, not approximations from a calculator.
(a) $\lim _{x \rightarrow \frac{1}{4}} \cos (\pi x)$
(b) $\lim _{x \rightarrow 3} \frac{x^{2}-9}{x^{2}+2 x-15}$
(c) $\lim _{t \rightarrow 2} \frac{\frac{1}{2}-\frac{1}{t}}{2-t}$
(d) $\lim _{x \rightarrow \infty} \frac{1+3 x-2 x^{2}}{6 x^{2}-4 x+5}$
5. (12 points) Given $f(x)=\sqrt{2 x}$, find $f^{\prime}(x)$ using the definition of the derivative. Do not use any shortcuts to find the derivative. Show work.
6. (11 points) The function $g(x)=\frac{2}{x}$ has as its derivative

$$
g^{\prime}(x)=-\frac{2}{x^{2}}
$$

Use this information to find the equation of the tangent line at $x=3$. Show work.
7. (9 points) Decide whether each of the following three statements is true or false. Write the entire word TRUE or the entire word FALSE, and explain your answer. You must justify your answer with an explanation. Note that drawing a picture may help you explain your answer.
(a) If a function $f$ is discontinuous at a point $x=a$, then $f$ is not defined at $x=a$.
(b) Let $f$ and $g$ be two functions that are related by the fact that the graph of $g$ is obtained by shifting the graph of $f$ up three units. Then $f^{\prime}(x)=g^{\prime}(x)$ for all $x$.
(c) By the Intermediate Value Theorem, the function $f(x)=\frac{3 x^{2}-12 x+12}{x-2}$ has a root in the interval $(1,3)$.
8. (12 points) The graph shows the position $s$ (in kilometers) of a car $t$ hours into the trip.

(a) Is the car going faster at point $B$ or point $C$ ? Explain.
(b) What is the car doing between point $D$ and point $E$ ? Explain.
(c) What was the average velocity for the entire trip (point $A$ to point $F$ )?

Bonus Find the value of $k$ so that the limit

$$
\lim _{t \rightarrow-1} \frac{t^{2}+k t-3}{t+1}
$$

exists. What is the value of the limit? Show work.

