Suppose that \( \{v, w\} \) is a linearly independent set in a vector space \( V \).
Prove that \( \{v - w, 3v + 2w\} \) is a linearly independent set as well.

Since \( \{v, w\} \) is a linearly independent set,
if \( a(v) + b(w) = 0 \) then \( a = 0 \) and \( b = 0 \).

Now consider the set \( \{v - w, 3v + 2w\} \).

If \( c_1(v - w) + c_2(3v + 2w) = 0 \) \( (*) \)

I find \( c_1, c_2 \).

Regrouping the left hand side:

\[
(c_1 + 3c_2)v + (-c_1 + 2c_2)w = 0
\]

By lin ind of \( \{v, w\} \), we have

\[
c_1 + 3c_2 = 0 \quad \text{and} \quad -c_1 + 2c_2 = 0
\]

\[
\begin{bmatrix}
-1 & 3 & 1 & 0 \\
-2 & 1 & 0 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 0 & 0 \\
0 & 5 & 0 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

So \( c_1 = 0 \) and \( c_2 = 0 \).

Since the only solution to equation \( (*) \) above is the trivial solution, the set \( \{v - w, 3v + 2w\} \) is linearly independent.