Spring 2021 Math 220 Test 2 Name:__________________

1. Use Mathematical Induction to prove that for all natural numbers n

$$\sum_{k=1}^{n}(3k-2) = 1+4+7++(3n-2) = \frac{n(3n-1)}{2}$$

2. Use Mathematical Induction to prove that $n! > 4^n$ for all natural numbers $n \geq 4$.

3. The Fibonacci numbers are defined by $f_1 = 1, f_2 = 1$ and for each natural number n, $f_{n+2} = f_{n+1} + f_n$. Prove that for all natural numbers n, the Fibonacci number f_{3n} is even.

4. Make a Venn diagram that illustrates that for any sets A and B, $A \cap B \subseteq A$ and $A \subseteq A \cup B$. Then prove these assertions.

5. Prove that for any sets A and B, $(A \cap B)^c = A^c \cup B^c$.

6. Let $A = \{x, y\}$ and let $B = \{1, 2\}$ and let S be their Cartesian product, i.e., $S = A \times B$. List the elements of S. How many elements are in the power set of S, $P(S)$? Is $S \subseteq P(S)$, or is $S \in P(S)$?

7. For each natural number n, let $A_n = \{x \in \mathbb{R} \,|\, n-1 < x < n\}$. Prove that $\{A_n \,|\, n \in \mathbb{N}\}$ is a pairwise disjoint family of sets.

8. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$. Sketch the graph of f. Prove that f is neither one-to-one nor onto. How could the domain and codomain be modified to make f a bijection?