8. 1. Find $\frac{dy}{dx}$ by implicit differentiation. 

$$x^2 - 3y^2 = 9 + 2xy.$$ 

Assume $y$ is a function of $x$. 
($y = f(x)$)

$$\frac{d}{dx} (x^2 - 3y^2) = \frac{d}{dx} (9 + 2xy)$$

$$2x - 6y \cdot \frac{dy}{dx} = 2x \cdot \frac{dy}{dx} + 2y$$

Chain rule

$$2x - 6y = 2x \frac{dy}{dx} + 6y \frac{dy}{dx} = (2x + 6y) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x - 6y}{2x + 6y} = \frac{\sqrt{x-9}}{x+3y} = \text{slope}$$

5. 2. Find an equation of the tangent line to the curve $x^2 - 3y^2 = 9 + 2xy$ at the point $(3, 0)$. 
(Note that it is the same curve in #1 above.)

$$y - y_0 = m(x - x_0)$$

($x_0, y_0) = (3, 0)$

So

$$y - 0 = m(x - 3)$$

Now

$$m = \frac{dy}{dx} \Big|_{(3, 0)}$$

Use answer to part (a).

$$m = \frac{3-0}{3+3(0)} = 1$$

and

$$y - 0 = 1(x - 3)$$

$\therefore y = x - 3$
3. Find the derivatives of the following functions. You do not need to simplify your answer.

(a) \( f(x) = x^4 + 4^x \)
\[
f'(x) = 4x^3 + 4^x \cdot \ln(4) \quad b/c \quad \frac{d}{dx} (a^x) = a^x \cdot \ln(a)
\]

(b) \( f(x) = \ln(x \cos x) \)
\[
\frac{d}{dx} (x \cos x) = \frac{1}{x \cos x} \left( x(-\sin x) + \cos x \right) \quad (\text{quotient rule})
\]
\[
\text{(chain rule)}
\]

(c) \( f(x) = \left(\frac{x^3 - 2x}{5x^2 + 3}\right)^4 \)
\[
4 \left(\frac{x^3 - 2x}{5x^2 + 3}\right)^3 \cdot \frac{d}{dx} \left(\frac{x^3 - 2x}{5x^2 + 3}\right) \quad \text{(chain rule)}
\]
\[
= 4 \left(\frac{x^3 - 2x}{5x^2 + 3}\right)^3 \cdot \left[\frac{(5x^2 + 3)(3x^2 - 2) - (x^3 - 2x)(10x)}{(5x^2 + 3)^2}\right] \quad \text{(quotient rule)}
\]

4. An object is launched vertically up on Planet X. The position, \( s(t) \) (in feet), of the object after \( t \) seconds is given by
\[
s(t) = 60t - 12t^2
\]

(a) What is the maximum height reached by the object?
\[
v(t) = s'(t) = 60 - 24t = 0
\]
\[
\Rightarrow t = \frac{60}{24} = 2.5 \text{ seconds}
\]
\[
\text{Max height} = s(2.5) = 60(2.5) - 12(2.5)^2
\]
\[
= 150 - 12(6.25) = 150 - 75 = 75 \text{ ft.}
\]

(b) How fast (i.e., find velocity) is the object moving when it is 48 feet on the way up?
\[
s(t) = 48 \Rightarrow 60t - 12t^2 = 48
\]
\[
\Rightarrow t^2 - 5t + 4 = 0 \Rightarrow (t-1)(t-4) = 0
\]
\[
\Rightarrow t = 1 \text{ sec} \quad \text{up} \quad t = 4 \text{ sec} \quad \text{down}
\]
\[
V(1 \text{sec}) = 60 - 24(1) = 36 \text{ ft/sec}.
\]
5. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$. Answer the following using calculus. (Answer produced from graphing calculator receives no credit.) Answer in the space provided.

(a) Find the intervals on which $f$ is increasing or decreasing.

\[ f'(x) = 6x^2 - 6x - 12 = 0 \]
\[ x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \]
\[ x = 2, -1 \]

\[ f' > 0 \quad f' < 0 \quad f' > 0 \]

\[ -1 \quad 2 \]

$f$ is increasing on $(-\infty, -1)$ and $(2, \infty)$; $f$ is decreasing on $(-1, 2)$

(b) Find the $x-$values where $f$ attains its local maximum and minimum values.

$f$ has local maximum at $x = -1$.  
$f$ has local minimum at $x = 2$.

6. Consider the function $f(x) = 2x^3 - 3x^2$. Answer the following using calculus. (Answer produced from graphing calculator receives no credit.) Answer in the space provided.

(a) Find the intervals on which $f$ is concave up or concave down.

\[ f''(x) = 12x - 6 = 0 \]
\[ x = \frac{1}{2} \]

\[ f'' < 0 \quad f'' > 0 \]

\[ -\frac{1}{2} \quad \frac{1}{2} \]

$f$ is concave up on $(-\infty, \frac{1}{2})$; $f$ is concave down on $(-\frac{1}{2}, \infty)$

(b) Find the $x-$coordinate(s) of inflection point(s) of $f$.

\[ f \text{ has inflection point(s) at } x = \frac{1}{2} \]
7. A girl flies a kite at a height of 500 ft, the wind carrying the kite horizontally away from her at a rate of 15 ft/sec. How fast must she let out the string when the kite is 1300 ft away from her? Assume the string is taut so that it forms a straight line.

\[
\frac{dz}{dt} = 15 \text{ ft/sec}
\]

Find \( \frac{dz}{dt} \) when \( z = 1300 \) ft.

Let \( x^2 + 500^2 = z^2 \), where \( x \) and \( z \) are functions of \( t \).

\[
2x \cdot \frac{dx}{dt} = 2z \cdot \frac{dz}{dt}
\]

When \( z = 1300 \) ft, \( x = 1200 \) ft, by Pythagorean theorem.

So \( 2(1200)(15) = 2(1300) \frac{dz}{dt} \)

And \( \frac{dz}{dt} = \frac{(1200)(15)}{1300} = \frac{1800}{13} \text{ ft/sec} \)

8. If \( y = f(x) \) is a function such that \( f' > 0 \) for all \( x \) and \( f'' > 0 \) for all \( x \), which of the following could be part of the graph of \( y = f(x) \)? (Choose one, no partial credit.)
9. Find the absolute maximum and absolute minimum values of the function

\[ f(x) = x^3 - 6x^2 + 20 \]

in the interval \([0.5, 5]\).

\[ f'(x) = 3x^2 - 12x \text{ exists everywhere} \]

\[ f'(x) = 0 \implies 3x(x-4) = 0 \implies x = 0, 4 \]

\[ f(0.5) = 18.625 \]
\[ f(4) = -12 \]
\[ f(5) = -5 \]

\[ \text{Absolute max } \frac{\sqrt{3}}{3} \text{ at } x = 0.5 \]
\[ \text{and absolute min } -12 \text{ at } x = 4 \]

10. Sketch a graph of the function \( f \) that satisfies the following conditions. Label all intercepts, local maxima/minima and inflection points, if any.

(a) \( f(-3) = 0, \ f(0) = 2, \ f(1) = 0, \ f(4) = 0 \).

(b) \( f'(-1) = 0, \ f'(3) = 0 \).

(c) \( f' > 0 \) on the intervals \((-\infty, -1) \cup (3, \infty)\), and \( f' < 0 \) on the interval \((-1, 3)\).

(d) \( f'' < 0 \) on the interval \((-\infty, 1)\) and \( f'' > 0 \) on the interval \((1, \infty)\).
Circle the correct answer. You do not need to show your work. (No partial credit will be given.)

3. 11. The position of a particle moving on a straight line is \( s(t) = \sec t \), its acceleration at \( t = \pi/4 \) is given by

(a) \( 2\sqrt{2} \)  (b) \( 2 + 2\sqrt{2} \)  (c) \( \frac{\pi}{2} \)  (d) \( 3\sqrt{2} \)  (d) \( 4\sqrt{2} \)

\[ v(t) = s'(t) = \sec(t) + \tan(t) \quad \text{and} \quad a(t) = v'(t) = \sec(t) \cdot \sec^2(t) + \tan(t) \cdot \sec(t) \]

So, \( a\left(\frac{\pi}{4}\right) = \sec^3\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right) \tan^2\left(\frac{\pi}{4}\right) \)

3. 12. If \( f(x) = \sin^2 x \), its derivative, \( f'(x) \), is given by

(a) \( \cos^3 x \)  (b) \( \sin^3 x \)  (c) \( \cos^2 x \)  (d) \( -2\sin x \cos x \)  (c) \( 2\sin x \cos x \)

\[ f'(x) = 2\sin(x) \cdot \cos(x) \quad \text{chain rule} \]

3. 13. The derivative of \( y = x^x \) is given by

(a) 1  (b) \( x \cdot x^{-1} \)  (c) \( x \ln x + x^x \)  (d) \( x^x (1 + \ln x) \)  (e) \( xe^x + \ln x \)

\[ g'(x) = g(x) \cdot \frac{d}{dx} \ln(g(x)) = x^x \left( 1 + \ln(x) \right) \]

3. 14. The number of tablets and smartphones in use worldwide (in millions) in year \( t \) from 2010 through 2012 is approximately \( f(t) = 128 t^{1.9} \), \( 1 \leq t \leq 3 \) where \( t = 1 \) corresponds to 2010. The rate at which the number of tablets and smartphones is changing (in millions per year) in 2011 is given by

(a) 486.40  (b) 453.83  (c) 477.71  (d) 256  (d) 0

\[ f'(t) = 128 \left( 1.9 \right) t^{0.9} \]

\[ f'(1) = 128 \left( 1.9 \right) \cdot 1^{0.9} \approx 453.83 \]

\[ f'(2) = 128 \left( 1.9 \right) \cdot 2^{0.9} \approx 453.83 \]